Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

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Uni. Roll No. ....

Program/ Course:B.Tech.
Semester : 3
Name of Subject: Engineering Mathematics-II1
Subject Code: BSEC-101
Paper ID: 16030

Time Allowed: 03 Hours **Max. Marks: 60**

NOTE:

1) **Part – A & B are compulsory**
2) **Part- C** has two Questions Q8 & Q9 and both are compulsory, but with internal choice.
3) Any missing data may be assumed appropriately.

Part – A **[Marks: 02 each]**

Q1.

a) Find Laplace transform of t^2e^{-3t}.

b) Find $f'(z)$, if $f(z) = x^2 - y^2 - 2xy + i(x^2 - y^2 + 2xy)$ is analytic.

c) If $a = cis\alpha, b = cis\beta, c = cis\gamma$ then prove that $\frac{ab}{c} + \frac{c}{ab} = 2\cos(\alpha + \beta - \gamma)$.

d) Separate real and imaginary parts of $\sin(x + iy)$.

e) Write Bessel's and Legendre's differential equations .

f) Solve the following partial differential equation:

$$\frac{\partial^2 z}{dx^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$$

Part – B **[Marks: 04 each]**

Q2. Find Laplace transform of $\frac{e^t - cost}{t}$.

Q3. Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic.

Q4. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$; where $J_n(x)$ is Bessel's function of order n.

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Q5. Solve the following partial differential equation:

$$\frac{\partial^2 z}{dx^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$$

Q6. If $x + iy = \cosh(u + iv)$, show that $x^2 sec^2 v - y^2 cosec^2 v = 1$.

Q7. If $i^{\alpha+i\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$.

Part – C **[Marks: 12 each]**

Q8 Using Laplace transform, solve the following differential equation:

$$\frac{d^3y}{dt^3} + 2\frac{\partial^2 y}{dt^2} - \frac{dy}{dt} - 2y = 0,$$ where y(0) = 1 , y'(0) = 2 and y''(0) = 2 .

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3cos\theta}$ using Residue theorem .

Q9. A tightly stretched flexible string has its ends fixed at x = 0 and x = l. At time t = 0 the string is given a shape defined by y(x,0) = $Asin\frac{\pi x}{l}$ and then released . Show that the displacement of any point at a distance x from one end at time t is given by y(x,t) = $Asin\frac{\pi x}{l} cos\frac{\pi ct}{l}$

OR

(a) Find the indicial equation of the differential equation :

$2x(1-x)\frac{d^2y}{dx^2} + (5-7x)\frac{dy}{dx} - 3y = 0.$

(b) Solve the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = sinx\ siny$ by direct integration, given that $\frac{\partial z}{\partial y} = -2siny$ when x = 0 and z = 0 when y is an odd multiple of $\frac{\pi}{2}$.

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